# Parameterization of the Meridional Eddy Heat and Momentum Fluxes

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#### ABSTRACT

Green's eddy diffusive transfer representation is used to parameterize the meridional eddy heat flux. The structural function obtained by Branscome for the diagonal component  $K_{yy}$  in the tensor of the transfer coefficients is adopted. A least squares method that uses the observed data of eddy heat flux is proposed to evaluate the magnitude of  $K_{yy}$  and the structure of the nondiagonal component  $K_{yz}$  in the transfer coefficient tensor. The optimum motion characteristic at the steering level is used as a constraint for the relationship between  $K_{yy}$  and  $K_{yz}$ . The obtained magnitude of  $K_{yy}$  is two to three times larger than that of the Branscome's, which is obtained in a linear analysis with the assumption of  $K_{yz} = 0$ .

Green's vertically integrated expression for the meridional eddy momentum flux is used to test the coefficients obtained in the eddy heat flux. In this parameterization, the eddy momentum flux is related to the eddy fluxes of two conserved quantities: potential vorticity and potential temperature. The transfer coefficient is taken to be the sum of that obtained in the parameterization of eddy heat flux, plus a correction term suggested by Stone and Yao, which ensures the global net eddy momentum transport to be zero. What makes the present method attractive is that, even though only the data of eddy heat flux are used to evaluate the magnitude of the transfer coefficients, the obtained magnitude of the eddy momentum flux is in good agreement with observations. For the annual mean calculation, the obtained peak values of eddy momentum flux are 94% of the observation for the Northern Hemisphere and 101% for the Southern Hemisphere. This result significantly improves the result of Stone and Yao, who obtained 34% for the Northern Hemisphere and 16% for the Southern Hemisphere in a similar calculation, but in which  $K_{vs} = 0$  was assumed.

## 1. Introduction

The purpose of this paper is to investigate the use of classic eddy transfer theory to parameterize eddy heat and momentum fluxes. An adequate parameterization of the eddy heat and momentum fluxes is extremely important for successfully simulating the mean climate fields, such as the zonal wind and temperature, in a twodimensional zonally averaged climate model. The eddy heat flux plays a major role in transporting heat from low latitudes to high latitudes and maintaining the energy balance of the atmosphere, and the eddy momentum flux is of critical importance for determining the surface zonal winds and the Ferrel cells. A widely used method for the parameterization of eddy fluxes is the diffusive transfer representation, also known as the mixing-length theory of fluid particles, which relates the eddy flux linearly to the gradient of the mean field, with the correlation coefficients called transfer coefficients. The transfer coefficients are positive in the initial stages of amplifying waves (Green 1970; Plumb 1979).

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Some successes were achieved in parameterizing the eddy heat flux using the mixing-length method (Green 1970; Branscome 1983; Stone and Yao 1990; Genthon et al. 1990). Unfortunately, for the eddy momentum transport the simple diffusive transfer representation is inappropriate because negative transfer coefficients are required. This "negative viscosity" (see MacCracken and Ghan 1988) can be attributed to the momentum not being a conservative quantity.

Green (1970) first proposed to derive the eddy momentum flux from the diffusive transports of two approximately conservative quantities: potential temperature and quasigeostrophic potential vorticity. However, Green did not give an explicit expression for the transfer coefficients. In later studies, White (1977), White and Green (1984), and Wu and White (1986) studied how different schemes of transfer coefficients would affect model solutions of the general circulation and surface zonal flow. In those studies the latitudinal variations of the transfer coefficients were specified. It was found that model solutions depend on the detailed spatial specification of the transfer coefficients. A promising scheme was given by Branscome (1980), in which the transfer coefficient depended on some dynamic constants derived from the approximate solution of the Charney model of baroclinic instability. Stone and Yao (1987)

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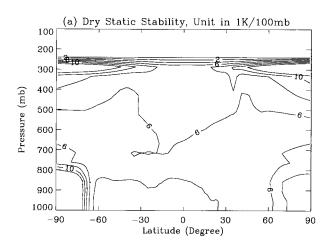
explicitly calculated the eddy momentum flux using Green's method by using the transfer coefficient obtained by Branscome (1980). However, their results indicated that the parameterization greatly underestimated the eddy momentum flux. Their results for January calculation showed that the peak values of eddy momentum flux by the parameterization for a dry model atmosphere were only 34% in the Northern Hemisphere and 16% in the Southern Hemisphere of that obtained by a general circulation model (GCM) simulation. The result is contrary to the findings of White (1977) and Wu and White (1986) who obtained strong surface zonal winds for a different set of transfer coefficients.

Because of the underperformance of their parameterization scheme for a dry atmosphere, Stone and Yao (1987; hereafter SY87) proposed a parameterization scheme for a moist atmosphere. With this moist scheme, they obtained a peak flux of 75% of that obtained by GCM simulations. In this moist scheme, the increase of the eddy momentum flux is mostly caused by using an "effective" moist static stability (eddy momentum transport increases as the static stability decreases). The moist static stability is smaller than the dry static stability due to the inclusion of the latent heat release. Specifically, the moist static stability in SY87, denoted by  $\sigma_m$ , is expressed as

$$\sigma_{m} = -\left[\frac{\partial \overline{\theta}}{\partial p} + \frac{L}{c_{p}} \left(\frac{p_{0}}{p}\right)^{\kappa} \frac{\partial \overline{q}}{\partial p}\right]$$

$$\approx -\left[\frac{\partial \overline{\theta}}{\partial p} + \overline{\text{rh}} \frac{L}{c_{p}} \left(\frac{p_{0}}{p}\right)^{\kappa} \frac{\partial q_{s}(\overline{T})}{\partial p}\right], \tag{1}$$

where  $\theta$  is the basic-state potential temperature, p the pressure,  $p_0$  the pressure at the surface, L the latent heat constant,  $c_n$  the specific heat,  $\overline{q}$  the specific humidity of the basic state,  $q_s(T)$  the saturated specific humidity relative to the basic-state temperature T, rh the relative humidity of mean state, and  $\kappa$  the ratio of gas constant to  $c_p$ . Note that the  $\sigma_m$  definition in (1) is opposite in sign to the original definition in SY87 in order for a stable atmosphere to have positive static stability. For application under real atmospheric conditions, there are two major drawbacks in using  $\sigma_m$  as a parameter for the parameterization of eddy momentum flux. One is that Eq. (1) fails to describe moisture effects correctly when rh approaches zero. This is because the moist parameterization scheme should have an influence on the eddy momentum transport only when latent heat release occurs due to condensation. When rh is small, for example, 0.2, as in the desert latitudes, there are practically no clouds and precipitation in the atmosphere. In this situation, the momentum transport should use the dry static stability  $(-\partial \overline{\theta}/\partial p)$ , but Eq. (1) still gives a decreased static stability and, as a consequence, leads to an increased eddy momentum transport. In other words, the SY87 moist scheme does not provide a realistic transition from dry atmosphere to moist atmosphere.



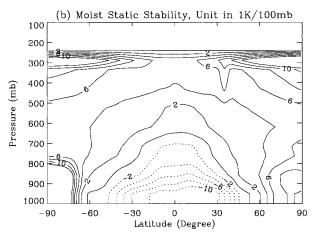


FIG. 1. Static stability distribution of (a) dry atmosphere and (b) moist atmosphere, in  $10^{-2}$  K mb<sup>-1</sup>. The annual mean temperature and specific humidity data are taken from Oort and Peixóto (1983).

The second shortcoming of (1) is that it may lead to negative static stability at lower and middle latitudes. Figures 1a,b show comparisons between the dry static stability and the moist static stability calculated using observed annual mean data. In Fig. 1 the temperature and specific humidity data are taken from Oort and Peixóto (1983). It can be seen that the dry static stability is quite uniform throughout the whole troposphere; however, in the lower atmosphere of the lower and middle latitudes, due to the higher water vapor concentration, the moist static stability becomes negative. Salustri and Stone (1983) obtained similar results for observed January data. In the SY87 model atmosphere the moist static stability in the Tropics and subtropics remained positive, so no problem occurred. However, the problem may emerge if the convection in a model is not strong enough to maintain an absolutely stable model atmosphere.

The above difficulties in using the SY87 moist scheme suggest that we still need a technique that will work for the real atmosphere. In this study we attempt to develop such a scheme. The transfer coefficient  $K_{vv}$ 

used in the SY87 scheme was obtained by tacitly assuming that the second transfer coefficient  $K_{yz}$  was zero [see Eq. (2) below]. In this study we will show that by properly parameterizing both transfer coefficients  $K_{yy}$ and  $K_{vz}$ , the parameterized meridional eddy heat and momentum transports can agree reasonably well with observations even for a dry atmosphere, suggesting it is not necessary to invoke the moist process in the parameterization. In this study, the observed eddy heat flux will be used to evaluate the magnitude of  $K_{yy}$  and the structure of  $K_{yz}$ . A least squares method with a constraint on the trajectory slope characteristic at the steering level is proposed to obtain  $K_{yy}$ . The constraint is similar to one of the schemes described in White and Green (1984). The inclusion of  $K_{yz}$  greatly increases the estimation of the magnitude of  $K_{yy}$ . With the proposed parameterization, the obtained transfer coefficient  $K_{vv}$  is two to three times larger than that used in SY87. The larger coefficient causes the peak value of the calculated eddy momentum transport for annual mean data to increase three to four times compared to the SY87 case and agrees well with observations. This result suggests that the large underestimation for the dry atmospheres in the SY87 calculation may be attributed to the lack of influence of the  $K_{vz}$  term.

In the next section the detailed derivation and results of the parameterization of the eddy heat flux will be given. Section 3 describes the parameterization of the eddy momentum flux and shows calculated results with different transfer coefficients. Section 4 discusses the validation and limitations of the parameterization.

## 2. Parameterization of eddy heat flux

a. Green's (1970) scheme and Branscome's (1983) scheme

Green (1970) showed that, because the potential temperature is approximately conserved during the baroclinic development, the eddy heat flux can be represented as

$$\overline{v'\theta'} = -K_{yy}\frac{\partial\overline{\theta}}{\partial y} - K_{yz}\frac{\partial\overline{\theta}}{\partial z}, \qquad (2)$$

where the overbar represents the zonal mean, the prime represents the deviation from the zonal mean, v is the meridional velocity,  $\theta$  the potential temperature, and  $K_{yy}$  and  $K_{yz}$  the transfer coefficients in the horizontal and vertical directions, respectively. The spatial variation of the transfer coefficients can be estimated from the solution of an appropriate baroclinic instability problem. Green (1970) had performed such calculations and discussed the general behavior of the coefficients in a baroclinic atmosphere, especially near the steering level. Green's calculation indicated that at the steering level an air parcel would move along the optimum orientation, which is at half the slope of the isentropic surface, so the coefficients would have the following relationship:

$$\frac{K_{yz}}{K_{yy}} = \frac{\overline{v'\Delta z}}{\overline{v'\Delta y}} = -0.5 \frac{\frac{\partial \overline{\theta}}{\partial y}}{\frac{\partial \overline{\theta}}{\partial z}}.$$
 (3)

Branscome (1983) reviewed the method and dynamics that had been used to represent the eddy heat flux and developed a parameterization similar to the diffusive transfer expression. In his parameterization the eddy heat flux is related to the mean potential temperature gradient field and some dynamic constants obtained from the baroclinic instability analysis. His parameterization is expressed as

$$\overline{v'\theta'} = A_0 \frac{gN_s}{\theta_s f^2} d^2 \left(\frac{\partial \overline{\theta}}{\partial y}\right)^2 e^{-z/d_K},\tag{4}$$

where  $N_s$  is the Brunt–Väisällä frequency;  $\theta_s$  the basic-state potential temperature; g the gravity constant; f the Coriolis parameter;  $A_0$  a nondimensional constant; and d and  $d_K$  are vertical scales defined by Branscome (1980) and Zou (1995),

$$d = \frac{H}{K_c} \tag{5}$$

and

$$d_K = \frac{H}{(4K_c^2 + 1)^{1/2} - 1},\tag{6}$$

respectively, where H is the density-scale height and  $K_c$  the nondimensional horizontal wavenumber of the most unstable Charney wave. Two representations for  $K_c$  had been proposed. Branscome's (1983) approximate formula for  $K_c$  is

$$K_c = \frac{1+\gamma}{2},\tag{7}$$

where  $\gamma$  is defined by

$$\gamma = \frac{\beta H N_s^2}{f^2 \frac{\partial \overline{u}}{\partial z}},\tag{8}$$

where  $\beta = df/dy$  is the  $\beta$  parameter and  $\partial \overline{u}/\partial z$  the vertical shear of zonal wind. Stone and Yao (1990) used another expression,

$$K_c = \left[ \left( \frac{1 + \gamma}{r_c} \right)^2 - \frac{1}{4} \right]^{1/2},$$
 (9)

where  $r_c = 1.35$ . Equation (9) is based on the accurate solution of the Charney model obtained by Wang et al. (1985). Stone and Yao (1990) noticed that Eq. (9) would cause the heat flux to be more concentrated toward the surface and the total heat transport decreased by about 15% as compared to using Eq. (7). We will discuss the difference between Eqs. (7) and (9) in more detail in

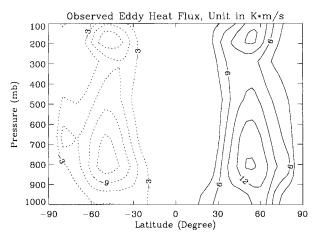


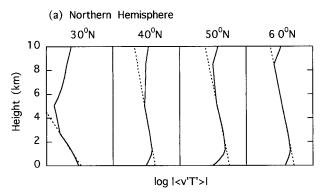
Fig. 2. Observed annual mean eddy heat flux. Data are from Oort and Peixóto (1983). Contour interval is 3 K m  $s^{\rm -1}$  and the zero line is not plotted.

the next section and revise Eq. (9) so that Eqs. (7) and (9) will compromise with each other.

Branscome's parameterization scheme provides reasonable results for the eddy heat flux compared to the observations. Also, it leads to specification of  $K_{yy}$  in terms of the mean potential temperature field and internal dynamic constants of a baroclinic wave. However, it ignores the influence of the  $K_{yz}$  term; therefore, the coefficient  $K_{yy}$  derived from it would be underestimated. In the following sections, we propose a modified method for the parameterization, which is a combination of Green's method and Branscome's approach. The results will be comparable to Branscome's scheme. By the proposed method, we can obtain a transfer coefficient that can be used in the parameterization of eddy momentum flux.

#### b. Vertical scale of the eddy heat flux

As mentioned in the last section, Eq. (9) leads to the parameterized eddy heat flux to be confined to a much shallower height than the observed and thus significantly underestimates the total eddy heat flux. The reason is that a different expression of the most unstable wave results in a different vertical scale  $d_K$ . This vertical scale determines the depth of the important transport and thus the total transport of eddy heat. Because, in the single-wave approximation,  $d_{\kappa}$  is related only to the most unstable wave, the selection of the most unstable wave should be such that  $d_K$  is close to the observation. For this purpose, the derived vertical scale from observations is used to determine the most unstable wave in this paper. The observed eddy heat flux data are taken from Oort and Peixóto (1983) and are shown in Fig. 2. Note that the data shown are total eddy heat flux, that is, stationary plus transient fluxes. Stone (1984) argued that there is a strong negative feedback between stationary and transient eddy heat fluxes, so a parameter-



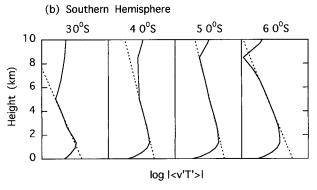


FIG. 3. Vertical log  $|\overline{v'T'}|$  profiles (solid lines) and their corresponding linear decrease (dotted lines) at different latitudes for the (a) Northern Hemisphere and (b) Southern Hemisphere. The range of  $|\overline{v'T'}|$  is from 0 to 4 for each latitude. The  $\overline{v'T'}$  data are taken from Oort and Peixóto (1983).

ization may work better for the total flux rather than an individual component. Figures 3a,b show several examples of the log vertical profiles of the eddy heat flux and their corresponding linear decrease in the middle troposphere at different latitudes of the Northern Hemisphere and Southern Hemisphere, respectively. Figure 4 shows the corresponding latitudinal variation of the observed vertical wave scale. As Figs. 3 and 4 indicate, the vertical scale is larger in the midlatitudes, and it decreases toward the Tropics. This is consistent with Branscome's argument that the vertical scale decreases as  $\gamma$  increases. Between 15°N and 15°S the eddy heat flux is too small so it is difficult to obtain reliable vertical scales from the observation. No observed vertical scale is shown in this region. At higher latitudes, the vertical scale is smaller than at midlatitudes. This is unexpected because the scale discussion of Branscome (1980) would predict the vertical scale to be close to the density scale at high latitudes. The reason for this inconsistency is not clear. It may be related to the lower tropopause at higher latitudes.

In Fig. 4 the vertical scales of the most unstable wave calculated by (7) and (9) for  $r_c = 1.35$  and  $r_c = 1.83$  are shown for comparison. It is seen that for the lower and middle latitudes of the Southern Hemisphere, Branscome's expression is quite close to the observa-

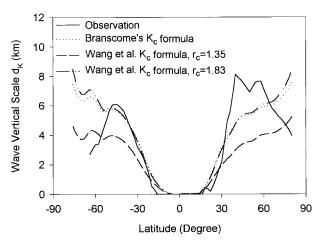


FIG. 4. Latitudinal variation of the observed and parameterized wave vertical scale  $d_{\kappa}$ . The solid line is derived from the observed annual mean  $\overline{v'T'}$  data of Oort and Peixóto (1983). The dotted line is calculated from Branscome's  $K_c$  formula. The dashed and dotted—dashed lines are the parameterization using Wang et al.'s (1985)  $K_c$  formula, where  $r_c$  is taken to be 1.35 and 1.83, respectively.

tion. The expression of Wang et al. (1985) for  $r_c=1.35$  systematically gives a smaller vertical scale. However, when  $r_c=1.83$  is used in (9), the obtained vertical scales are also close to the observations. In (9),  $r_c$  is a measure of the wavelength. A larger  $r_c$  corresponds to longer waves. To see this more clearly, one can transform  $r_c$  into an integer zonal wavenumber. Using the definition  $K_c=\sqrt{k^2+l^2}$ , where k and l are zonal and meridional wavenumbers, respectively; and  $k=m/(a\cos\varphi_0)$ , where m is the integer zonal wavenumber, a the earth radius, and  $\varphi_0$  the midlatitude; and also assuming l is comparable with k as in Branscome (1980), that is, l=k, one obtains

$$m = \frac{1}{\sqrt{2}} \left[ \left( \frac{1 + \gamma}{r_c} \right)^2 - \frac{1}{4} \right]^{1/2} \frac{f_0}{N_s H} a \cos \varphi_0.$$
 (10)

For typical midlatitude conditions,  $f_0 = 10^{-4} \, \mathrm{s}^{-1}$ ,  $N_s = 0.01 \, \mathrm{s}^{-1}$ ,  $\gamma = 1$ , and  $H = 7.3 \, \mathrm{km}$ . When using  $r_c = 1.35$ , we find m = 6.1 and when  $r_c = 1.83$  we get m = 4.3. Figure 4 indicates that, for a better fit of the vertical scale, the representative wave for the parameterization should be chosen as m = 4.3, not the linear most unstable wave m = 6.1. This probably occurs because most of the eddy heat transport in the Southern Hemisphere is accomplished by the intermediate-scale waves m = 4 and 5 (Solomon 1997) so that the representative wave in the parameterization has to be located in this region.

In the following calculation, we fix  $r_c$  at 1.83 and use (9) to calculate the wavenumber of the representative wave for the parameterization of eddy heat flux. This choice leads to the same results for the wave's vertical scale as in (7). Notice that this choice does not give a good fit in the midlatitudes of the Northern Hemisphere. This is because almost half of the eddy heat transport

in the Northern Hemisphere is accomplished by the stationary eddies. There is no mechanism to describe the stationary eddies in the current parameterization, so we simply use  $r_c = 1.83$  for both the Southern Hemisphere and Northern Hemisphere, and the transient scheme is used to represent transient plus stationary eddies. This may not be a severe problem, as noted in Branscome (1983), when only the total eddy transport is considered. This is because both transient and stationary eddies compete for the same amount of available potential energy. Manabe and Terpstra (1974) showed that the total amount of transport from transient eddies only, when the topography was absent, was about the same as from both transient and stationary eddies when topography was present. However, because the topography may change the spectral structure of the eddy energy conversion and cause the energy conversion (thus energy transport) to move toward longer waves (Manabe and Terpstra 1974), the horizontal structure of the calculated eddy heat flux using the present parameterization for the Northern Hemisphere may be different from observations.

# c. Effects of the $K_{yz}$ term

Equation (4) is the parameterization of eddy heat flux derived from the Charney model and the single-wave approximation by Branscome (1983). Because in the Charney model  $N_s$  and  $\partial \overline{\theta}/\partial y$  are assumed to be constants, Branscome proposes that the calculation of  $\overline{v'\theta'}$  should be carried out in an all-term averaged way, that is,

$$\overline{v'\theta'} = A_0 \frac{g\langle N_s \rangle}{\langle \overline{\theta} \rangle f^2} d^2 \left\langle \left( \frac{\partial \overline{\theta}}{a \partial \varphi} \right)^2 \right\rangle e^{-z/d_K}, \tag{11}$$

where the brackets indicate a vertical average. Branscome suggests that the vertical average should be calculated by using  $e^{-z/d_K}$  as the weighting function:

$$\langle x \rangle = \frac{\int_0^\infty x e^{-z/d_K} dz}{\int_0^\infty e^{-z/d_K} dz}.$$
 (12)

Stone and Yao (1990) and Genthon et al. (1990) used (11) to calculate the eddy heat flux in their zonally averaged atmospheric models. The coefficient  $A_0$  is in a range between 0.15 and 0.25 (in the papers of SY87 and Genthon et al.,  $A_0$  is between  $4 \times 0.15$  and  $4 \times 0.25$ ; here the factor 4 is absorbed in the d formula) and the parameter  $\gamma$  is given by

$$\gamma = \frac{\beta \langle H \rangle \langle N_s^2 \rangle}{f^2 \langle \partial \overline{u} / \partial z \rangle}.$$
 (13)

Branscome's method provides a closure form for the parameterization of the eddy heat flux. However, be-

cause the transfer coefficients will be needed in the parameterization of the eddy momentum flux, it is necessary to generalize (4) to obtain an expression like (2), in which the eddy heat flux is related to the local gradient of the mean potential temperature. In order to obtain the generalized form, we can assume that in Eq. (4) the eddy heat flux is linearly related to the local gradient of the potential temperature (Branscome 1980); that is, only one of the  $\partial\theta/\partial\varphi$  factors is considered to be vertically averaged. By comparing (4) with (2) and assuming  $K_{vz}=0$  one can obtain the expression for  $K_{vv}$  as

$$K_{yy} = A_0 \frac{gN_s}{\theta_s f^2} \left\langle \left| \frac{\partial \overline{\theta}}{a \partial \varphi} \right| \right\rangle d^2 e^{-z/d_K}. \tag{14}$$

The expression (14) is exactly the same as that used in SY87. From the above generalization process of (4), we cannot obtain a separate expression for  $K_{vz}$  (actually,  $K_{yz} = 0$  has been assumed). So  $K_{yz}$  has to be obtained by a different method. In the following, by using the observed eddy heat flux data, a least squares fit with a constraint on the trajectory slope characteristics at the steering level is used to obtain an approximate expression for  $K_{yz}$ . The scheme in this paper is similar to one of White and Green's (1984). It follows Green's argument that far away from boundaries the particle displacements tend to be along the optimum direction. An optimum direction is where an air parcel can obtain maximum available potential energy so that maximum kinetic energy and wave growth result. Eady (1949) and Green (1960) proved that the optimum orientation was one-half the slope of the isentropic surface. Green (1970) found in his numerical calculation that the optimum orientation was nicely attained near the steering level. According to the optimum direction argument, Green derived Eq. (3) for the relationship between  $K_{yz}$ and  $K_{yy}$  at the steering level. Equation (3) is not valid where the boundary effect has significant influence on the motions. At the earth's surface, vertical velocity and displacement are small because of rigid conditions. At the tropopause, vertical displacement is also small due to stratification, so  $K_{yz} \rightarrow 0$  at these two boundaries (Green 1970). Combining these boundary conditions and Eq. (3) we can assume  $K_{yz}$  has the following form:

$$K_{yz} = -p_n(z') \left(\frac{\partial \overline{\theta}}{\partial y}\right) \left(\frac{\partial \overline{\theta}}{\partial z}\right)^{-1} K_{yy},$$
 (15)

where z' = z/D, D is a vertical scale chosen to be  $d_K$  in this paper,  $p_n(z')$  is a polynomial of z' to the order of n,

$$p_n(z') = \sum_{i=1}^n B_i z'^i,$$
 (16)

and  $B_i$  is a coefficient to be determined by least squares fit of the parameterization to the observation. The polynomial  $p_n(z')$  should be zero at the surface and tropopause and 0.5 at the steering level. White and Green

(1984) assumed  $p_n(z')$  to be independent of latitude and to take the form of a parabola in the vertical direction. Here  $p_n(z')$  is an implicit function of latitude because  $d_K$  has latitudinal variation.

Introducing (14) and (15) into (2) one obtains the final form of the parameterization:

$$\overline{v'\theta'} = -A_0[1 - p_n(z')]\zeta(\varphi, z), \tag{17}$$

where  $\zeta(\varphi, z)$  is defined by,

$$\zeta(\varphi, z) = \frac{g\langle N_s \rangle}{\langle \overline{\theta} \rangle f^2} d^2 e^{-z/d\kappa} \left\langle \frac{\partial \overline{\theta}}{a \partial \varphi} \right\rangle \frac{\partial \overline{\theta}}{a \partial \varphi}.$$
 (18)

Equation (17) contains n+1 unknown constants  $A_0$ ,  $B_1, \ldots, B_n$ . These constants can be obtained by a least squares fit of (17) to observations. The structural function  $\zeta(\varphi, z)$  is determined only by wind and temperature observations, while  $A_0$  and  $p_n(z')$  can be different for different n. This is an important feature of the parameterization that will have significant consequences on the eddy momentum parameterization. In the following, we discuss the results of this parameterization.

The data used are annual mean observations taken from Oort and Peixóto (1983) as shown in Fig. 2. The annual mean temperature and zonal wind data are also taken from Oort and Peixóto (1983). In the following calculations, we have used a cutoff exponential function,  $[1 - \exp(-z/\Delta z)]$ , suggested by Stone and Yao (1990), to dampen the flux maximum in the planetary boundary layer. The numerical simulation of Branscome et al. (1989) indicates that this dampening is due to the surface friction and heat flux. The surface friction restricts the growth of the meridional velocity variance while the surface heat flux restrains the growth of the temperature variance near the ground. These processes cause the maximum eddy heat transport to be located at 850 mb, rather than at the surface. Stone and Yao (1990) used  $\Delta z = 450$  m. Here  $\Delta z = 550$  m is used, which is more suitable to the current dataset.

The major impact of including  $p_n(z')$  in the parameterization is that when a least squares method is used to evaluate  $A_0$ , the obtained  $A_0$  will be two to three times larger than that used by previous authors. We consider three cases to discuss this feature.

The first case is to assume  $p_n(z')=0$  (equivalent to n=0). The least squares method results in an  $A_0$  of 0.37 for the Northern Hemisphere and 0.27 for the Southern Hemisphere. The Northern Hemisphere value is 100% and the Southern Hemisphere value 50% larger than that given by Branscome (1980), which is obtained by a linear analysis and closure assumption (in Branscome's case,  $A_0=1/4\sqrt{2}=0.18$  as used in SY87).

Figure 5 shows the calculated eddy heat flux for the case of n = 0. Compared to Fig. 2, the calculated peak values at the lower troposphere are almost the same as the observed. The total calculated eddy heat transport is 62% of the observation for the Northern Hemisphere and 76% for the Southern Hemisphere. Notice that in

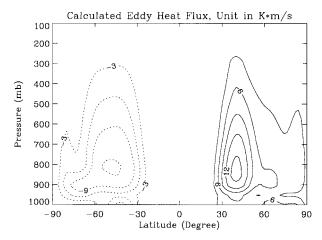


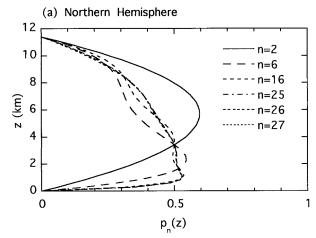
Fig. 5. Calculated eddy heat flux by Eq. (17) for case n=0. Contour interval is 3 K m s<sup>-1</sup> and the zero line is not plotted.

the parameterized field the secondary maximum near the tropopause in the observation has been missed, which is the major reason for the underestimation of the parameterization. The total transport above 300 mb contributes 24% for the Northern Hemisphere and 16% for the Southern Hemisphere for the observed annual mean data. This secondary maximum may be caused by the persistent disturbances of planetary waves in the stratosphere, which are not accounted for in this parameterization.

The second case is the ideal (but impossible) situation in which all particle motions are in the optimal direction, and so  $p_n(z') \equiv 0.5$ . In this case, values of  $A_0$  for the Northern and Southern Hemispheres are just twice the values obtained in the first case: 0.74 and 0.54, respectively. The calculated eddy heat fluxes are, of course, unchanged.

The third case is the more realistic situation in which  $p_n(z')$  is forced to be zero at the surface and tropopause and 0.5 at the steering level. In the midlatitudes  $d_K \approx$ 5.7 km, so we choose an upper boundary  $z_T' = 2$  (or  $z_T$ =  $2 \times d_K$ ), corresponding to a tropopause height of 11.4 km there. This upper boundary depends on latitude due to  $d_{\kappa}$ . It is roughly consistent with the observed tropopause at the middle and high latitudes but approaches zero at low latitudes, which ensures the obtained eddy heat flux to be small at low latitudes because of finite z' and  $p_n(z')$ . Branscome's (1983) analysis indicates that for fast-growing Charney modes the steering level  $z'_s$  is between 0.5 and 0.8. In this study,  $z'_s$  is fixed at 0.6. Sensitivity experiments (not shown in the paper) indicate that the results presented in the following will not be significantly influenced by the different choice of  $z'_s$  as long as  $z'_s$  is in the range between 0.5 and 0.8.

The calculation is performed for n=2 through 27 (n=1 does not satisfy the imposed boundary conditions, so it is not considered here). Figures 6a,b show some examples of  $p_n(z')$  profiles for the Northern Hemisphere and the Southern Hemisphere, respectively.



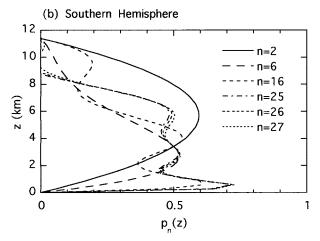
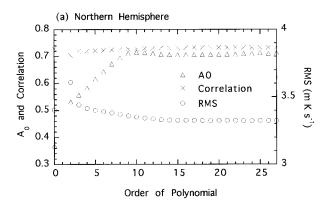


Fig. 6. The  $p_n(z')$  profiles for different n for the (a) Northern Hemisphere and (b) Southern Hemisphere, where  $z=z'\times 5.7$  km.

When n becomes larger, the Northern Hemispheric profiles converge quite well. In the lower and upper troposphere, the Southern Hemispheric profiles also converge well. However, in the middle troposphere, the profiles do not converge as well as in the Northern Hemisphere. Higher-order polynomials may be needed to better represent the behavior of the Southern Hemisphere profiles. At this point, it seems that  $p_n(z')$  is approaching 0.5 in the entire middle troposphere of the Southern Hemisphere.

Figures 7a,b show the behavior of the correlation and root-mean-square (rms) error between the observation and parameterization as the polynomial order increases for the Northern Hemisphere and Southern Hemisphere, respectively. The behavior of  $A_0$  is also shown in these figures. The worst parameterization occurs when n = 2. As n increases, the correlation gradually increases from its minimum value at n = 2. The rms has similar behavior to the correlation except it is decreasing as n increases. When n is large enough (approximately larger than 10), the correlation will be larger and rms smaller than that of n = 0. Thus, based on the evaluation of



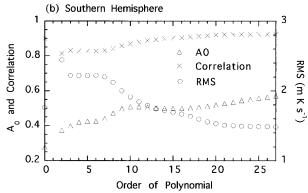


Fig. 7. Variations of  $A_0$  (triangles), correlation (crosses), and rms (circles) between observation and parameterization vs the order of polynomial; (a) Northern Hemisphere; (b) Southern Hemisphere.

the correlation and the rms, any parameterization of  $n \ge 10$  is as good as or better than n = 0.

For comparison with the case n = 0 (Fig. 5), Fig. 8 shows the calculated eddy heat flux for n = 27. In this case, the total calculated eddy heat transport is 64% of the observation for the Northern Hemisphere and 80% for the Southern Hemisphere. Figures 9a,b show comparisons between the observation and the parameterization for n = 0 and n = 27 of the vertically and hemispherically averaged  $\overline{v'T'}$ , respectively. In Fig. 9a the peak values of the vertically averaged eddy heat fluxes are 70% of the observed in both hemispheres. The observed vertical mean below 300 mb (so the tropopause maximum is excluded) is also shown for comparison. The overall comparison between observation and parameterization is fairly good for the Southern Hemisphere, but there is deficit for the parameterization at the middle and higher latitudes for the Northern Hemisphere. We believe that this deficit is due to the lack of long-wave contribution in the parameterization. As mentioned earlier, Manabe and Terpstra (1974) indicated that topography in the Northern Hemisphere caused the maximum eddy energy conversion to occur at zonal wavenumber 2 or 3. Because the parameterization in this paper is based on wavenumber 4.3 at the midlatitude, and in the Northern Hemisphere almost half of the total eddy transport is caused by the stationary eddies,

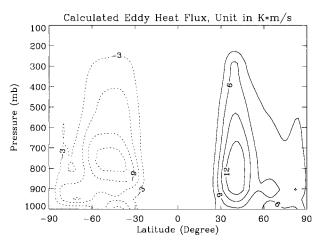
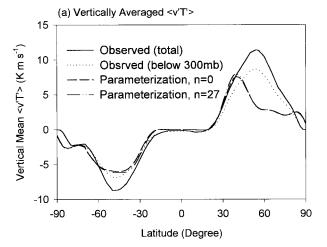


Fig. 8. Same as Fig. 5 except for parameterization of n = 27.



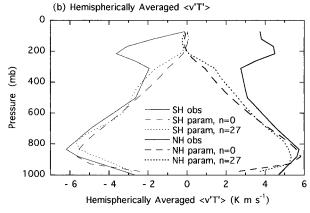


FIG. 9. Comparison between observation and parameterization of (a) vertically averaged  $\overline{v'T'}$  and (b) hemispherically averaged  $\overline{v'T'}$ , where observation is from Oort and Peixóto (1983). (a) The solid line is the observation integrated over the entire vertical height and the dotted line the integration below 300 mb. The dashed line is the parameterization of n=0 and the dotted—dashed line the parameterization of n=27. (b) The solid lines indicates observation, dashed lines for parameterization of n=27. The thick lines are for the Northern Hemisphere and thin lines for the Southern Hemisphere.

the deficit in the higher latitudes is an inevitable consequence. It appears to be necessary to include the longer waves in any parameterization in the future, especially for the Northern Hemisphere where the stationary eddies are important.

For the comparison between n=0 and n=27 cases, Fig. 9b indicates that the calculated positions of the maximum transport at the lower troposphere are closer to the observed for the n=27 case and the overall fit below 300 mb is improved compared to the n=0 case, especially in the Southern Hemisphere. For the Southern Hemisphere, the rms for the n=27 case decreased by 16% compared to the n=0 case. No matter how large n becomes, the correlation (or rms) cannot reach 1 (or 0) because the horizontal structure of the parameterization is largely determined by the structural function  $\zeta(\varphi, z)$ , which is not affected by n.

At n = 27,  $A_0 = 0.71$  for the Northern Hemisphere and 0.57 for the Southern Hemisphere. Similar to the case of  $p_n(z') \equiv 0.5$ , the values of  $A_0$  for n = 27 are almost doubled compared to n = 0 case. This occurs because most of the eddy heat transports occur in the lower and middle troposphere (the tropopause secondary maximum is not accounted for in this parameterization) where  $p_{27}(z')$  is very close to 0.5. As a consequence, the average effect is that  $A_0$  is close to the value of the  $p_n(z') = 0.5$  case. This feature allows parameterization by simply choosing  $p_n(z') \equiv 0.5$ . As seen in the above comparisons, there is no significant difference in the eddy heat flux structure between the  $p_{27}(z')$  and  $p_n(z') = 0$  cases. The primary impact of choosing  $p_{27}(z')$ is to allow  $A_0$  to be about twice as large as the case with  $p_n(z') = 0$ . As will be seen in the next section, doubling  $A_0$  allows the eddy momentum flux parameterization to be closer in magnitude to observations. Therefore, using  $p_n(z') \equiv 0.5$  will work equally well as using a high-order polynomial for either the eddy heat flux or the momentum flux.

### 3. Parameterization of eddy momentum flux

#### a. Green's scheme

Green's parameterization is based on the relationship between eddy heat, momentum, and quasigeostrophic potential vorticity that is expressed as

$$-\frac{1}{\cos^2\varphi}\frac{\partial(\overline{u'v'}\cos^2\varphi)}{\partial\theta} = \overline{v'\eta'} + f_0\frac{\partial}{\partial p}\frac{\overline{v'\theta'}}{\sigma}, \quad (19)$$

where  $\sigma = -\partial \overline{\theta}/\partial p$  is the dry static stability and  $\eta$  is the quasigeostrophic potential vorticity. Because  $\eta$  is approximately conserved following the horizontal direction of baroclinic motions, the eddy flux of  $\eta$  can be expressed as (Green 1970)

$$\overline{v'\eta'} = -K_{yy} \frac{\partial \overline{\eta}}{\partial y}.$$
 (20)

Equations (19) and (20) have been used by Wiin-Nielsen

and Sela (1971) to evaluate  $K_{yy}$  from observations of eddy heat and momentum transport. In this paper  $K_{yy}$  is evaluated using only the observed eddy heat flux, so (19) and (20) can be used to test whether or not the obtained  $K_{yy}$  is appropriate for the eddy momentum flux.

Only the vertically integrated eddy momentum flux is considered here. By assuming a horizontally uniform static stability, substituting Eqs. (2) and (20) and the quasigeostrophic vorticity definition

$$\overline{\eta} = -\frac{1}{\cos\varphi} \frac{\partial (\overline{u} \cos\varphi)}{a\partial\varphi} + f - f_0 \frac{\partial}{\partial\rho} \frac{\overline{\theta}}{\sigma}$$

into (19), and integrating the resulting equation over pressure, one obtains the spherical polar version of Green's expression for the eddy momentum flux,

$$-\frac{1}{\cos^{2}\varphi}\frac{\partial}{a\partial\varphi}\int_{0}^{p_{s}}\overline{u'v'}\cos^{2}\varphi dp$$

$$=\int_{0}^{p_{s}}\left\{-K_{yy}\frac{\partial}{a\partial\varphi}\left[-\frac{1}{\cos\varphi}\frac{\partial(\overline{u}\cos\varphi)}{a\partial\varphi}+f\right]\right.$$

$$\left.-\frac{f_{0}}{\sigma}\frac{\partial\overline{\theta}}{a\partial\varphi}\frac{\partial K_{yy}}{\partial\rho}\right\}dp. \tag{21}$$

The first term at the right-hand side of (21) is called the barotropic term and the second term the baroclinic term (White 1977). These two terms are of the same order of magnitude but of opposite signs and the result of the summation is a smaller net convergence or divergence of the eddy momentum flux, depending on the latitude. Green (1970), as well as other researchers (e.g., White 1977; White and Green 1984; Wu and White 1986), applied (21) to construct simple models to study the behavior of the surface zonal flow and general circulation. White and Green (1984) tested various schemes for representing  $K_{yy}$  and  $K_{yz}$  and studied how these schemes affect the model results. The steeringlevel constraint between  $K_{yy}$  and  $K_{yz}$  in this paper is similar to one of their schemes, but in their study, the atmosphere was in a  $\beta$  plane. For spherical atmospheres, White (1977) obtained unrealistically strong surface zonal flow. It is further expected (White 1977) that (21) may be more accurate when the constant  $f_0$  is replaced by  $f = 2\Omega \sin(\varphi)$ , where  $\Omega$  is the earth's angular velocity, even though this is not strictly consistent with the conservation requirements of quasigeostrophic dynamics. White (1977) and Wu and White (1986) both found stronger surface zonal flow in this situation. On the contrary, Stone and Yao (1987) obtained too small a magnitude of eddy momentum flux (implying weak surface zonal flow) using (21) in a zonally averaged atmospheric model. In the SY87 calculation,  $f_0$  was also replaced by f, but different transfer coefficients were used. These different results indicate that detailed and realistic constructions of transfer coefficients are very important for obtaining realistic results. In the following, the same calculation in SY87 is repeated using observed annual mean data. Then, the newly obtained  $K_{yy}$  will be used for calculating eddy momentum flux. In the following calculation,  $f_0$  in (21) is replaced by f as in SY87 and the influence of this replacement will be discussed and compared with Wu and White's (1986) results. In this paper no attempt is made to apply (21) in climate models. Instead, the calculated results are directly compared with observations as well as the SY87 results. Zou (1995) applied the current parameterization scheme in a zonally averaged climate model and obtained general circulation and zonal surface flow with reasonable accuracy.

# b. Result with different $K_{yy}$

The divergence of the transport of eddy angular momentum must vanish upon global integration. We assume that v'=0 at the equator, so that (when weighted by  $\cos^2\varphi$ ) the right-hand side of (21) should also vanish in the hemispheric integral. In order to satisfy this requirement, we adopt the SY87 treatment. In their treatment, the following modification to the Branscome's transfer coefficient has been introduced:

$$K'_{yy} = K_{yy} + K_{NL},$$
 (22)

where  $K_{NL}$  is a correction to the coefficient  $K_{yy}$  that is meant to allow for modifications of the wave structure associated with the mature stage of the life cycle of a baroclinic instability and to increase divergence of the eddy momentum flux in the Tropics and subtropics to balance the convergence at the midlatitudes. Typical values of  $K_{NL}$  are an order of magnitude smaller than  $K_{yy}$ . Stone and Yao provide the following construction for  $K_{NL}$ :

$$K_{NL} = \begin{cases} K_0 & [\overline{u}] > 0\\ K_0 e^{-y_0/L} & [\overline{u}] < 0, \end{cases}$$
 (23)

where  $K_0$  is a constant to be obtained by setting the integral of (21) over the hemispheres to be zero, the bracket indicates the vertical mean,

$$[\overline{u}] = \frac{1}{p_s} \int_0^{p_s} \overline{u} \, dp, \tag{24}$$

 $y_0$  is the distance to the region of easterlies from the latitude where  $[\overline{u}] = 0$ , and L is a characteristic decay scale in the evanescent region that is selected to be 626 km.

The above construction of  $K_{NL}$  plus the coefficient  $K_{yy}$  ensures the global net transport of eddy momentum to be zero. Note that this treatment differs from that of Green (1970), White (1977), White and Green (1984), and Wu and White (1986) who used the global net transport constraint to alter the rate of change of  $K_{yy}$  with height rather than  $K_{yy}$  itself.

In SY87, the peak values of the parameterized eddy momentum flux for January were only 34% in the Northern Hemisphere and 16% in the Southern Hemi-

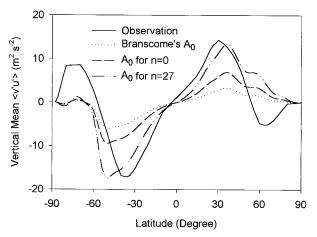


FIG. 10. Comparisons of  $\overline{u'v'}$  between observation and parameterization with different coefficient  $A_0$ . The solid line is the observation taken from Oort and Peixóto (1983). The dotted line is the parameterization with Branscome's  $A_0=0.18$ . The dashed line is the parameterization of n=0 where  $A_0=0.37$  for the Northern Hemisphere and 0.27 for the Southern Hemisphere. The dotted–dashed line is the parameterization of n=27 with  $A_0=0.71$  for the Northern Hemisphere and 0.57 for the Southern Hemisphere.

sphere compared to a GCM simulation. The same calculation is repeated in this paper using the annual mean data of temperature and zonal wind from Oort and Peixóto (1983). The results are compared with the observed annual mean eddy momentum flux also taken from Oort and Peixóto (1983). Figure 10 shows the comparisons between the observation and calculations. For Branscome's value of  $A_0=0.18$ , the calculated peak fluxes are 23% of the observed for the Northern Hemisphere and 35% for the Southern Hemisphere. This result is quite close to the SY87 result. Because their result is for a January simulation and in this month there is more baroclinic activity in the Northern Hemisphere, the higher value for the Northern Hemisphere in January is consistent with our lower value for the annual mean case

When the newly obtained  $K_{yy}$  for n=27 is used, the peak values of eddy transport are 94% of the observed for the Northern Hemisphere and 101% for the Southern Hemisphere. The result for this case is also shown in Fig. 10. Compared to the Branscome case, this result indicates that the present parameterization can capture the eddy momentum transport by just using the dry atmospheric model without invoking the moist processes. Considering that only the observations for the eddy heat flux have been used to evaluate the transfer coefficients, the results are quite encouraging.

In order to see the net effect of including  $K_{yz}$  in the parameterization of eddy heat flux, we perform another calculation with the obtained  $A_0$  when  $K_{yz}=0$ . In this case  $A_0=0.37$  for the Northern Hemisphere and 0.27 for the Southern Hemisphere. The calculated peak values of eddy momentum flux are 49% of the observed for the Northern Hemisphere and 55% for the Southern

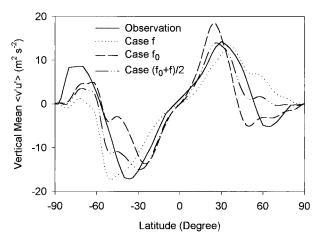


Fig. 11. Effect of the latitudinal variation of the Coriolis parameter on the parameterization. The solid line is the same observation as in Fig. 10. Parameterization is for n=27. The dotted line is for  $f=2\Omega\sin(\varphi)$ , same as in Fig. 10. The dashed and dotted–dashed lines are parameterizations for  $f=f_0$  and  $f=0.5\times [f_0+2\Omega\sin(\varphi)]$ , respectively.

Hemisphere. The relative increases of the eddy momentum flux from the  $K_{yz}=0$  case to the  $K_{yz}\neq0$  case are 92% for the Northern Hemisphere and 84% for the Southern Hemisphere, while the relative increase of  $A_0$  from the  $K_{yz}=0$  case to the  $K_{yz}\neq0$  case is 92% for the Northern Hemisphere and 111% for the Southern Hemisphere. These results indicate that increases in  $A_0$  cause a relative increase in the magnitude of the parameterized eddy momentum transport by approximately the same amount. So, including  $K_{yz}$  in the parameterization of eddy heat fluxes can lead to substantial differences in the parameterization of eddy momentum fluxes.

#### c. Effects of the Coriolis parameter

White (1977) and Wu and White (1984) found that the treatment of the Coriolis parameter has a significant influence on the results. In the above calculation,  $f_0$  has been replaced by  $f = 2\Omega \sin(\varphi)$  in (21). In order to see the effect of this replacement, we set  $f_0$  to be a typical midlatitude value. The result with  $K_{yy}$  of n=27 is shown in Fig. 11. Compared to the  $f=2\Omega \sin(\varphi)$  case, the most significant change is the decreased eddy transport (implying weaker surface zonal wind) toward middle latitudes, and equatorward eddy transport occurs at high latitudes (implying polar easterlies). The reason for this is that when  $f \rightarrow f_0$ , the baroclinic term becomes smaller in higher latitudes and larger in lower latitudes. Thus, the total eddy momentum convergence is smaller and easterlies are likely to occur near the poles. This is consistent with Wu and White's (1986) results. In addition, the calculated maximum transport is located equatorward to the observation for  $f = f_0$  while it is poleward for  $f = 2\Omega \sin(\varphi)$ . A compromise is to set  $f = 0.5[f_0]$  $+ 2\Omega \sin(\varphi)$ ]. The result for this case is also shown in Fig. 11. In this case, the maximum transport position is closer to the observation, especially for the Northern Hemisphere.

#### 4. Discussion

The described parameterization of the eddy heat and momentum fluxes is based on the Green (1970) and Branscome (1983) schemes, which are derived from idealized models of the baroclinic instabilities at the midlatitudes. Even though the parameterization is based on approximations of linear instabilities and a single wave, it produces many observed features for the eddy heat flux and vertically integrated eddy momentum flux. However, because only a single wave is used in the eddy heat flux parameterization, several observed features cannot be simulated by the scheme. First, the stationary eddies affect the structure of the eddy heat transport at the middle and higher latitudes in the Northern Hemisphere, which can be seen from the errors of either the vertical-scale fit or the structure of the calculated eddy heat flux. However, its parameterization remains an unsolved problem. Second, the tropopause secondary maximum in the observed eddy heat transport cannot be simulated by the scheme. Because of the lack of the secondary maximum in the parameterization, the peak value of the vertically integrated eddy heat transport is underestimated by 30% for both hemispheres. Third, the proposed method is to use the eddy heat flux data and the trajectory slope at the steering level as the constraints for the behavior of  $K_{yy}$  and  $K_{yz}$ . Therefore, if the steering level is outside the region under consideration or very close to the surface, then the proposed method for obtaining the vertical structure of  $K_{vz}$  will be invalid.

Contrary to the SY87 results, the transfer coefficients from the eddy heat flux parameterization in this paper provide good results for the vertically integrated eddy momentum flux for a dry atmosphere. This result suggests that it is not necessary to invoke a moist process in the eddy momentum flux parameterization. Furthermore, in the actual climate model applications, one can simply choose  $p_n(z') = 0.5$  for the parameterization. This works equally well as using the high-order polynomial for both eddy heat and momentum fluxes.

Because the final eddy momentum flux is obtained as the small difference between two large terms (baroclinic term minus barotropic term), the results are influenced by how to select the data to calculate those terms. In our calculation of the baroclinic term, the observed temperature data are used; in other words, the geostrophic zonal winds are used. If the thermal wind relation and the observed zonal wind data are used in calculating the baroclinic term, the resultant eddy momentum flux can be 20%–30% different from that presented here. The reason is that the geostrophic zonal wind is about 5%–10% larger than the observed zonal wind (Boville 1987). The error in the total eddy momentum flux can be amplified due to the large ratio of the baroclinic term to

the convergence of the total eddy momentum transport. For instance, if we assume that the ratio of the baroclinic term to the convergence of the eddy momentum flux is 4 and a 5% error is introduced in the baroclinic term, then the error of the total eddy momentum transport would be 20%. In order to rectify this problem, it appears that a data adjustment will be needed in future studies so that the data used will satisfy the thermal wind relation. In any case, the conclusion about the role of  $K_{yz}$  in our calculation will not be changed; that is, the eddy momentum flux in  $K_{yz} \neq 0$  case almost doubles that in the case  $K_{yz} = 0$ .

Finally, it is desirable to know how the obtained transfer coefficients affect other fluxes' parameterization, such as the vertical eddy heat flux and moisture fluxes developed by Stone and Yao (1990). Their scheme for vertical eddy heat flux is for a moist atmosphere and can be expressed as

$$\overline{w'\theta'} = \chi(r)\pi(z')\epsilon\overline{v'\theta'}, \qquad (25)$$

where  $\epsilon = -(\partial \overline{\theta}/\partial y)(\partial \overline{\theta}/\partial z)^{-1}$  is the slope of isentropic surface,  $\pi(z') = z' - 0.25z'^2$ , and  $\chi(r)$  is a correction factor accounting for the effect of moisture. The parameterization presented in this paper for  $\overline{v'\theta'}$  will not affect Stone and Yao's parameterization for  $\overline{w'\theta'}$ . As the key point in this paper indicates, including  $K_{yz}$  only changes the magnitude of  $K_{yy}$ , but not the magnitude of  $\overline{v'\theta'}$ . The eddy momentum flux is proportional to  $K_{yy}$ ; hence, it is affected by the details of the  $\overline{v'\theta'}$  parameterization. The vertical eddy heat flux is proportional to  $\overline{v'\theta'}$ , so it will not be influenced by the different choices of the transfer coefficients as long as  $\overline{v'\theta'}$  remains the same. The moisture fluxes parameterization in Stone and Yao (1990) are also proportional to the eddy heat flux, so they are still valid.

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